

# Flutter Prediction for Composite Wings Using Parametric Studies

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## Introduction

THE ability to control the stiffness (or flexibility) and cross-coupling properties to modify the free vibrational modes of beam-like composite structures, such as aircraft wings, has been widely reported in the literature (see, for example, Ref. 1). Further work to modify the aeroelastic stability of moderate-to-high-aspect-ratio lifting surfaces based on this concept can be found elsewhere.<sup>2,3</sup> Because flutter occurs due to the interaction between inertia and aerodynamic forces with structural deformations, i.e., elastic forces, it can be controlled by modifying the free vibrational characteristics using the concept.

The object of this Note is to investigate the flutter behavior of unswept composite wings when significant structural parameters are changed. Many of the previous investigators have studied the aeroelastic stability of laminated wings using the ply orientation as the main design variable.<sup>3,4</sup> However, there are notable exceptions in which rigidity properties have been used directly as design variables<sup>2,5,6</sup> instead of ply orientation. For instance, Austin et al.<sup>2</sup> use the ratio between the bending-torsion coupling rigidity and the bending rigidity ( $K/EI$ ) as a variable, although it is difficult to put upper and lower limits on this ratio. On the other hand, Niblett<sup>5</sup> uses two nondimensional ratios, one being the ratio between the bending and torsional flexibility and the other being the nondimensional flexibility cross-coupling parameter. In contrast, Weisshaar and Ryan<sup>6</sup> use rigidity-based nondimensional parameters in their theory, which are essentially the ratio between the torsional and bending rigidity ( $GJ/EI$ ) and the nondimensional cross-coupling parameter  $\psi$ , defined as  $\psi = K/(EIGJ)$ . [Note that the range for  $\psi$  is given by  $-1 < \psi < 1$  (Ref. 1).] The results given by Weisshaar and Ryan<sup>6</sup> are limited in the sense that they illustrate the variation of flutter and divergence speed for only three values of the ratio of torsional and bending rigidities. In this Note, however, the ratio between the uncoupled bending and torsional natural frequencies in the fundamental mode is taken as the variable instead of the ratio of the corresponding rigidities. This gives much wider applicability of results because it involves more parameters, such as mass, inertia, and length of the wing. Thus, it covers a broad range of variables when presenting the results. However, the nondimensional coupling parameter ( $\psi$ ) of Weisshaar and Ryan<sup>6</sup> has been retained in the analysis because it is dependent on the fiber orientation in a laminate and thus offers flexibility in design.

## Method of Analysis

The investigation has been carried out in the following stages. First, the rigidity properties of a composite wing are established using published theory.<sup>1,7</sup> Second, the free vibration characteristics are studied using the dynamic stiffness method.<sup>8</sup> Third, the flutter speed is computed by varying significant structural parameters, which include the cross-coupling parameter  $\psi$ , the ratio of the uncoupled fundamental bending and torsional natural frequencies

( $\omega_b/\omega_t$ ) and the static unbalance ( $x_\alpha$ ) expressed as a fraction of the semichord. (Note that  $x_\alpha$  is negative if the mass axis is behind the elastic axis, which is usually the case.) A well-established computer program called CALFUN<sup>9</sup> (Calculation of Flutter Speed Using Normal Modes), which uses generalized coordinates and normal modes (obtained from the dynamic stiffness method) together with Theodorsen unsteady aerodynamics in two-dimensional flow, was run to obtain the results. The nondimensional flutter speed  $V_F/b\omega_t$ , where  $V_F$  is the actual flutter speed,  $b$  is the semichord, and  $\omega_t$  is the (uncoupled) fundamental torsional natural frequency, is plotted against the above parameters. For a given range of input parameters, the maximum flutter speed obtained from the analysis was further checked by the well-known optimization program ADS.<sup>10</sup> The results presented are fairly general and apply to composite wings of any cross section as long as the rigidity and other properties are known within reasonable accuracy.

## Discussion of Results

Figure 1 shows the coordinate system and sign convention used in this Note, for a laminated composite beam (wing) together with the positive direction of the airflow. The results that follow apply to more general cross sections, but the rectangular cross section is shown in the figure for convenience.

Figures 2a, 2b, and 2c show the variation of the nondimensional flutter speed  $V_F/b\omega_t$  against the nondimensional uncoupled frequency ratio  $\omega_b/\omega_t$  for three different values of cross-coupling parameter  $\psi = 0, +0.4, -0.4$ , respectively. The first six normal modes of the cantilever wing were used in the analysis, which included both the fundamental bending and the torsional natural frequencies. The density ratio  $m/\pi\rho b^2$  and the nondimensional radius of gyration  $r_a = \sqrt{J_a/m b^2}$  were kept constant at 10 and 0.5, respectively, for all cases. The elastic axis location was assumed to be 20% of the semichord and forward of the midchord position ( $a = -0.2$ ). Several representative values of  $x_\alpha$  (the nondimensional distance between the elastic axis and the mass axis) were used to obtain the results, as shown in each set of figures. The results of Fig. 2a correspond to the degenerate case of metallic wing because the cross-coupling parameter  $\psi = 0$  in this case. These results generally agree with the results given in the classic text of Bisplinghoff et al.<sup>11</sup> for representative section of a rigid aerofoil [see Fig. 9.5(A), Graph (o), on p. 540 of Ref. 11]. The results of Figs. 2b and 2c, however, apply to composite wings. (Note that when the density ratio and the nondimensional radius of gyration were changed, trends similar to those of Fig. 2 were observed.) It is quite apparent from these figures that negative values of  $\psi$  give higher flutter boundaries (Fig. 2c) as opposed to the cases with positive values (Fig. 2b). This is in accord with observations made by other investigators<sup>3,4,6</sup> that the wash-in effect (negative  $\psi$  in this case) is generally beneficial from the flutter standpoint as it increases the flutter speed. The results of Fig. 2c, particularly for lower values of  $x_\alpha$  are helpful because they can be exploited to advantage to achieve higher (or even eliminate) flutter speeds.

Based on the above results, a representative value of  $x_\alpha = -0.1$ , which gave a suitable trend toward higher flutter speeds (see Fig. 2c), was chosen to obtain further results when the nondimensional frequency ratio is varied for different values of  $\psi$ . The negative values of  $\psi$  were used because they induce the so-called wash-in effect, which is beneficial for flutter. Figure 3 shows representative results where the nondimensional flutter speed  $V_F/b\omega_t$  has been plotted against the nondimensional frequency ratio for different values of negative  $\psi$ . Note that when the density ratio and the nondimensional radius of gyration were changed, trends similar to those of Fig. 3 were observed. The following observations can be made from this figure.

First, when  $\psi = -0.7$ , the nondimensional flutter speed increases rapidly for values of the nondimensional frequency ratio greater than 0.15, whereas it is almost invariant when this ratio is less than 0.15. Note, however, that because of large torsional rigidity ( $GJ$ ) usually associated with composite wings made of laminated flat beams (plates) or box beams,<sup>1,7</sup> the nondimensional frequency ratio is quite often very small, i.e., below 0.15. In such cases, any benefit that can be derived by changing  $\psi$  to increase the flutter

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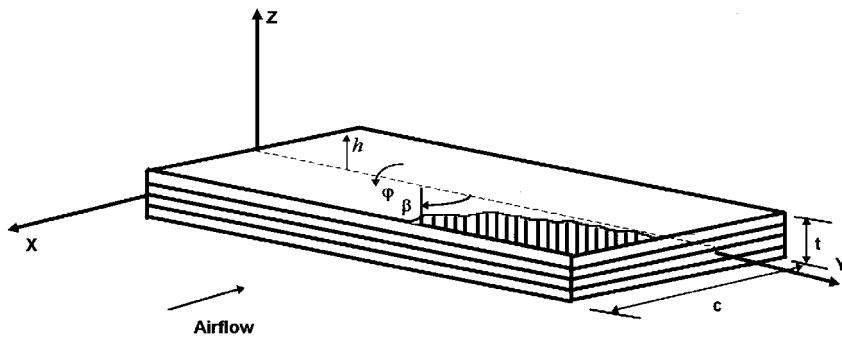
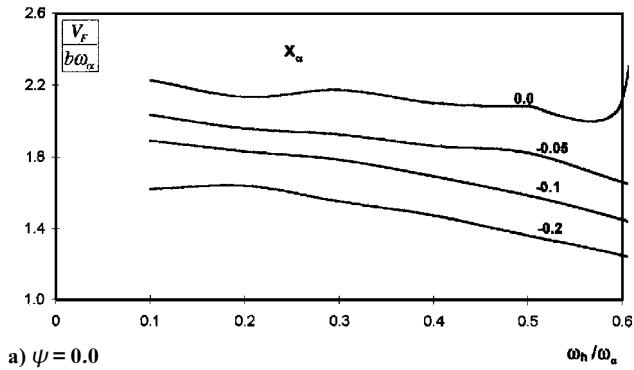
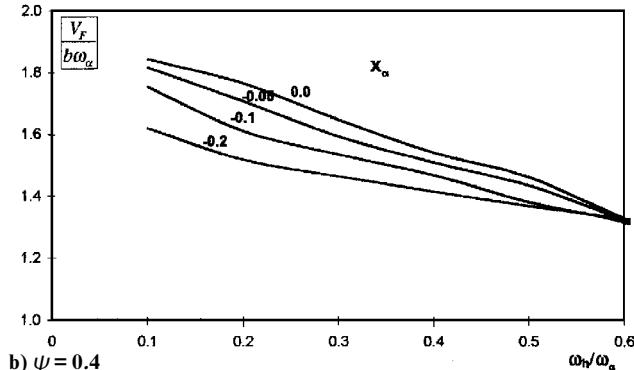
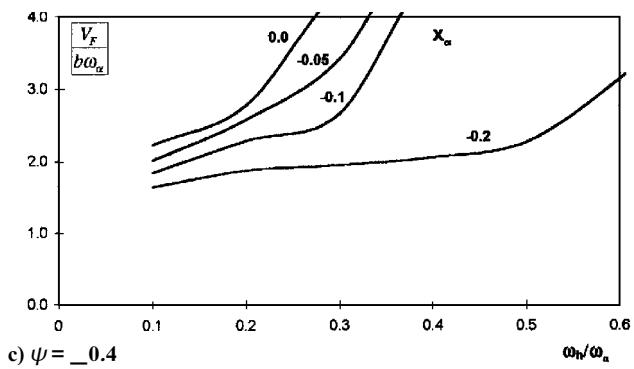
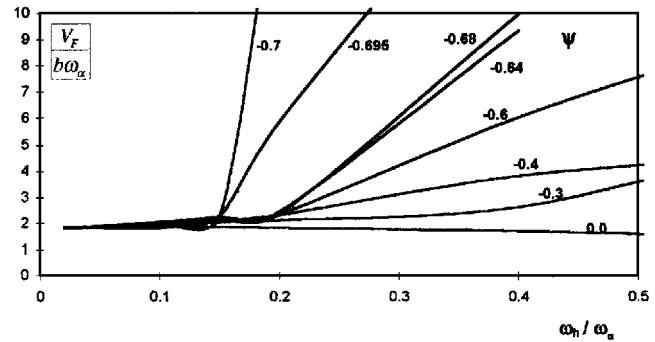


Fig. 1 Coordinate system and sign convention for a laminated composite beam.

a)  $\psi = 0.0$ b)  $\psi = 0.4$ c)  $\psi = -0.4$ Fig. 2 Dimensionless flutter speed  $V_F/b\omega_\alpha$  plotted against frequency ratio  $\omega_h/\omega_\alpha$  for various values of  $x_\alpha$  and  $\psi$  ( $m/\pi\rho b^2 = 10$ ,  $r_\alpha = 0.5$ ,  $a = -0.2$ ).

speed is quite marginal. In contrast, if  $GJ$  is relatively low but the coupling rigidity  $K$  is much higher, the graphs shown in Fig. 3 for values of the nondimensional frequency ratio above 0.15 can be very useful to achieve higher flutter speeds. Thus for values of nondimensional frequency ratio less than 0.15, maximizing  $GJ$  offers a better prospect of maximizing the flutter speed, as can be seen. Therefore, a laminate configuration that offers either the maximum  $GJ$  or the maximum negative  $\psi$  (i.e., minimum  $\psi$ ) gives the maximum flutter

Fig. 3 Dimensionless flutter speed  $V_F/b\omega_\alpha$  against frequency ratio  $\omega_h/\omega_\alpha$  for various values of cross-coupling parameter  $\psi$  ( $m/\pi\rho b^2 = 10$ ,  $r_\alpha = 0.5$ ,  $x_\alpha = -0.1$ ,  $a = -0.2$ ).

speed for an unswept wing. (Note that from a divergence point of view, the maximum negative  $\psi$  can be detrimental.<sup>6</sup>)

Three earlier and significant pieces of work<sup>3,4,6</sup> in this area are to be interpreted in relation to the present study as follows.

In Ref. 6, flutter was eliminated at a relatively low value of  $\psi$ . This is because the data used in Ref. 6 resulted in a high nondimensional frequency ratio  $\omega_h/\omega_\alpha$ , which was well above 0.15 (see Fig. 3), giving large variations in flutter speed with small changes in  $\psi$ . However, in Ref. 3, where a plate model based on lamination theory was used to predict the stiffness data, significant variation in flutter speed against the fiber orientation was observed, but this variation being shown in nondimensional form (i.e., it was shown relative to the flutter speed calculated when all ply angles were set to zero) did not reveal the independent effect of  $GJ$  on the flutter speed. Clearly the maximum value of the negative  $\psi$  did not eliminate flutter in Ref. 3. This is because the ratio  $\omega_h/\omega_\alpha$  was less than 0.15 (see Fig. 3). Furthermore, in Ref. 3 the computed flutter speed was not compared with the maximum achievable flutter speed from the same cross section of the laminate. Georghiades et al.,<sup>4</sup> however, made such a comparison. Their results revealed quite convincingly that for the same cross-sectional dimensions, the laminate configuration with maximum  $GJ$  gave much higher flutter speed than the one obtained using the maximum value of negative  $\psi$  for the particular laminate.

To validate the above explanation, an independent optimization study was carried out using the computer program ADS.<sup>10</sup> Two illustrative composite wings made of Hercules ASI/3501-6 graphite/epoxy material with cross-sectional properties chord  $c$ , thickness  $t$  (see Fig. 1), and static unbalance  $x_\alpha$  as 0.0762 m, 0.008 m, and  $-0.1$ , respectively, have been optimized to confirm the results. The only difference between the two wings is their lengths, which are 0.6 and 0.3 m, respectively.

Before the optimization studies were carried out, the nondimensional flutter speed ratio was computed for a wing with the same density ratio, nondimensional radius of gyration and static unbalance as given earlier. The results are shown in Fig. 4 for a wide range of nondimensional frequency ratios. Because the results of Fig. 4 are shown in nondimensional form, they apply to both wings of the illustrative examples. It is quite clear that when the nondimensional

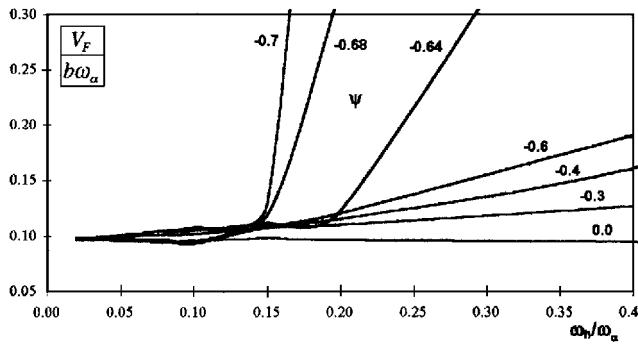
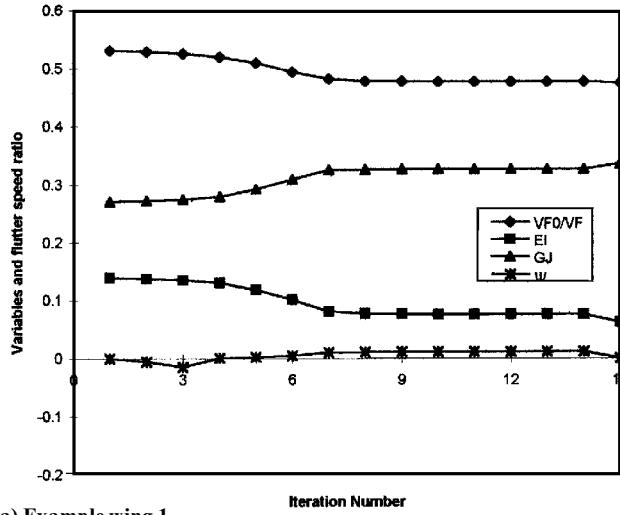
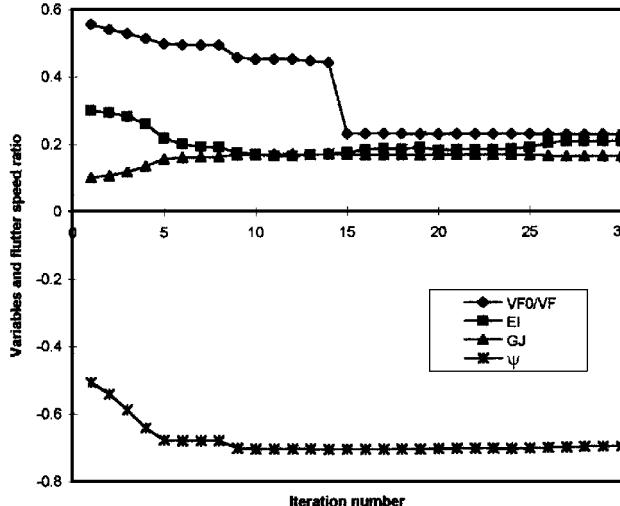


Fig. 4 Dimensionless flutter speed  $V_F/b\omega_a$  against frequency ratio  $\omega_b/\omega_a$  for various values of cross-coupling parameter  $\psi$  for the example wings 1 and 2 ( $m/\pi\rho b^2 = 16.67$ ,  $r_\alpha = 0.577$ ,  $x_\alpha = -0.1$ ,  $a = -0.2$ ).



a) Example wing 1



b) Example wing 2

Fig. 5 Aeroelastic tailoring history of example wings 1 and 2.

frequency ratio is below 0.15, the nondimensional flutter speed is almost invariant and is independent of  $\psi$ . However, note that, for the first example, wing of length 0.6 m, the nondimensional frequency ratio is always less than 0.15. So, the variation of  $\psi$  will not significantly alter the flutter speed for this wing but the maximum value of  $GJ$  will give the maximum flutter speed instead. This is confirmed by the results of optimization studies shown in Fig. 5a for this wing. The optimization was carried out for a number of different starting points of laminate configurations. Only the results

from the final computer run that gave the maximum flutter speed are shown in Fig. 5a. The variation of  $EI$ ,  $GJ$ ,  $\psi$ , and  $V_{FO}/V_F$  is plotted against the number of iterations.  $V_F$  is the computed flutter speed for a given stacking sequence, whereas  $V_{FO}$  is the flutter speed when all plies are set to 0 deg. As can be seen, the optimizer moves to maximum  $GJ$ , which gives the maximum flutter speed. This was predicted before from the analysis results shown in Figs. 3 and 4.

In the case of the second example of the wing with length of 0.3 m, the optimization results shown in Fig. 5b reveal a different picture. The reasons are as follows. First, the range of the nondimensional frequency ratio for this wing exceeds of 0.15 of Fig. 4, implying the predominant effect of  $\psi$ , which influences the flutter speed quite significantly. Thus, the optimizer in this case (see Fig. 5b) moves to the laminate configuration that maximizes the negative  $\psi$ , and so, in effect gives maximum flutter speed. Figure 5b shows the value of  $\psi$  to be about -0.7 when the flutter speed is optimized. This is in accord with the results shown in Figs. 3 and 4.

## Conclusions

The parametric investigation into the flutter characteristics of cantilevered composite wings in this Note shows that there are two very important parameters that affect the flutter speed of an unswept composite wing. These are 1) the ratio of the (uncoupled) fundamental bending and torsional natural frequencies ( $\omega_b/\omega_a$ ) and 2) the bending-torsion cross-coupling parameter  $\psi$ . The results show that when the ratio  $\omega_b/\omega_a$  is less than about 0.15, the maximum flutter speed can be achieved by maximizing the torsional rigidity  $GJ$ , whereas for higher ratios the maximum flutter speed can be achieved by obtaining the largest absolute value of the cross-coupling term  $\psi$ . Thus, the maximum flutter speed can be achieved either by maximizing  $GJ$  or negative  $\psi$ . This has been confirmed by independent optimization studies.

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